HIGHER-FOCK STATES AND THE COVARIANCE
OF THE LIGHT-FRONT QUARK MODEL

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The light-front quark model has been successful in describing various meson properties. We apply the model to the meson weak decays and discuss the covariance of the model. We find that the inclusion of the higher-Fock state(i.e. nonvalence or so called Z-graph) contributions is necessary to recover the covariance of the light-front quark model. We present the possibility of the effective calculation of the higher-Fock state contributions to restore the covarince in the light-front model.

Perhaps, one of the popular formulations for the analysis of such exclusive processes may be provided in the framework of light-front (LF) quantization. In particular, the Drell-Yan-West ($q^+ = q^0 + q^3 = 0$) frame has been extensively used in the calculation of various electroweak form factors and decay processes. As an example, only the parton-number-conserving (valence) Fock state contribution is needed in $q^+ = 0$ frame when the "good" component of the current, J^+ or $\mathbf{J}_\perp = (J_x, J_y)$, is used for the spacelike electromagnetic form factor calculation of pseudoscalar mesons. The LF approach may also provide a bridge between the two fundamentally different pictures of hadronic matter, i.e. the constituent quark model (CQM) (or the quark parton model) closely related to the experimental observations and the quantum chromodynamics (QCD) based on a covariant non-abelian quantum field theory. The crux of possible connection between the two pictures is the rational energy-momentum dispersion relation that leads to a relatively simple vacuum structure. There is no spontaneous creation of massive fermions in the LF quantized vacuum. Thus, one can immediately obtain a constituent-type picture, in which all partons in a hadronic state are connected directly to the hadron instead of being simply disconnected excitations (or vacuum fluctuations) in a complicated medium.

On the other hand, the analysis of timelike exclusive processes (or timelike $q^2 > 0$ region of bound-state form factors) remained as a rather significant challenge in the LF approach. In principle, the $q^+ \neq 0$ frame can be used to compute the timelike processes but then it is inevitable to encounter the particle-number-nonconserving

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Fock state (or nonvalence) contribution. The main source of difficulty in CQM phenomenology is the lack of information on the black blob in the nonvalence diagram arising from the quark-antiquark pair creation (see Fig. 1(a)).
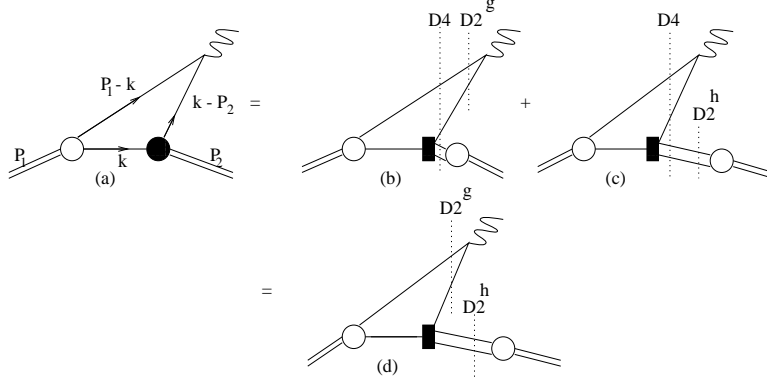

Fig. 1. Effective treatment of the light-front nonvalence diagram.

In this talk, we thus present the way of handling the nonvalence contribution. Our aim of new treatment is to make the program more suitable for the CQM phenomenology. As an application of our method, we investigate the semileptonic decay processes such as $K_{\ell 3}(\ell = e$ or $\mu)$ decays since the light-to-light decay processes bear the largest contribution from the nonvalence part. Also, the experimental data are best known in theses processes. Including the nonvalence contribution, our results not only show a definite improvement in comparison with the experimental data but also exhibit a covariance (i.e. frame-independence) of our model.

Thu current matrix element of the semileptonic pseudoscalar to pseudoscalar meson decays involve the two form factors:

$$J^{\mu}(0) = \langle P_2|\bar{Q}_2\gamma^{\mu}Q_1|P_1\rangle = f_+(q^2)(P_1 + P_2)^{\mu} + f_-(q^2)q^{\mu}, \tag{1}$$

where $q^{\mu} = (P_1 - P_2)^{\mu}$ is the four-momentum transfer to the lepton pair $(\ell\nu)$ and $m_{\ell}^2 \leq q^2 \leq (M_1 - M_2)^2$.

To illustrate our method [1], we treat the nonvalence state using the Schwinger-Dyson equation to connect the embedded-state shown as the black blob in Fig. 1(a) to the ordinary LF wave function (white blob in Fig. 1(d)). To make the program successful, we need some relevant operator connecting one-body to three-body sector shown as the black box in Fig. 1(d). The relevant operator is in general dependent on the involved momenta. Our **main observation** is that we can remove the four-body energy denominator D_4 using the identity $1/D_4 D_2^g + 1/D_4 D_2^h = 1/D_2^g D_2^h$ of the energy denominators(see Figs. 1(b) and 1(c)) and obtain the identical amplitude in terms of ordinary LF wave functions of photon and hadron (white blob) as shown in Fig. 1(d). For the small momentum transfer, perhaps the relevant operator may not

have too much dependence on the involved momenta and one may approximate it as a constant operator. In contact interaction case, we verified that our prescription of a constant operator in Fig. 1(d) is an exact solution of Fig. 1(a).

Table 1. Results for the parameters of $K_{\ell 3}^0$ decay form factors.

	$q^+ \neq 0$ frame		$q^+ = 0$ frame	Experiment [2]
	Effective(val + nv)	valence	valence	
$f_+(0)$	0.962	0.962	0.962	
λ_+	0.026	0.083	0.026	$0.0288 \pm 0.0015 [K_{e3}^0]$
λ_0	0.025	-0.017	0.001	$0.025 \pm 0.006 [K_{\mu3}^0]$
ξ_A	-0.013	-1.10	-0.29	$-0.11 \pm 0.09 [K_{\mu3}^0]$

In Table 1, we summarize the experimental observables for the $K_{\ell 3}$ decays, where $\lambda_i = M_\pi^2 f_i'(0)/f_i(0) (i = +, 0)$ and $\xi_A = f_-(0)/f_+(0)$. We use our linear potential parameters [3] in this analysis. As one can see in Table 1, our new results (column 2) for the slope λ_0 of $f_0(q^2) = f_+(q^2) + q^2 f_-(q^2)/(M_1^2 - M_2^2)$ at $q^2 = 0$ and $\xi_A = f_-(0)/f_+(0)$ are now much improved and comparable with the data. Especially, our result of $\lambda_0 = 0.025$ obtained from our effective calculation is in excellent agreement with the data, $\lambda_0^{\text{Exp.}} = 0.025 \pm 0.006$. We should note that the form factor $f_+(q^2)$ obtained from J^+ in $q^+ = 0$ frame is immune to the zero-mode contribution but the form factor $f_-(q^2)$ obtained from $J_\perp$ receives zero-mode contribution, i.e. the differences of λ_0 and ξ_A between our effective solutions and the $q^+ = 0$ frame results indicate the zero-mode contributions.

In summary, we presented an effective treatment of the LF nonvalence contributions crucial in the timelike exclusive processes. Using a SD-type approach and summing the LF time-ordered amplitudes, we obtained the nonvalence contributions in terms of ordinary LF wavefunctions of gauge boson and hadron that have been extensively tested in the spacelike exclusive processes. Including the nonvalence contribution, our results show a definite improvement in comparison with experimental data on $K_{\ell 3}$ decays. The frame-independence of our results also indicate that a constant relevant operator is an approximation appropriate to the small momentum transfer processes. Applications to the heavy-to-light decay processes involving large momentum transfers would require an improvement of the relevant operator including the momentum dependence. Consideration along this line is underway.

Acknowledgements

1. C. -R. Ji and H. -M. Choi, hep-ph/0009281.
2. Particle Data Group, D. E. Groom et al., Eur. Phys. J. C 15 (2000) 1.
3. H. -M. Choi and C. -R. Ji, Phys. Rev. D 59 (1999) 074015; Phys. Rev. D 59 (1999) 034001; Phys. Lett. B 460 (1999) 461.